

Black-Hole X-Ray Transients: The Effect of Irradiation on Time-Dependent Accretion Disk Structure

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(Received 1998 June 24; accepted 1999 April 25)

Abstract

Some effects of irradiation on time-dependent accretion-disk models for black hole X-ray novae are presented. Two types of irradiation are considered: direct irradiation from the inner hot disk and indirect irradiation as might be reflected by a corona or chromosphere above the disk. The shadowing effect of the time-dependent evolution of the disk height and consequent blocking of the outer disk by the inner and middle portions of the disk from the direct irradiation is included. The direct irradiation of the disk by inner layers where the soft X-ray flux is generated is found to have only a small effect on the outer disk because of shadowing. Mild indirect irradiation that flattens, but otherwise does not affect the light curve substantially, still has interesting non-linear effects on the structure of the disk as heating and cooling waves propagate. The irradiated disks do not always make simple transitions between the hot and cold states, but can linger at intermediate temperatures or even return temporarily to the hot state, depending on the irradiation and the activity in adjacent portions of the disk.

Key words: Accretion disks — Black holes — Instabilities — Stars: individual (A0620–00)— Stars: X-ray

1. Introduction

There is now broad recognition that the most common form of black-hole binary candidates in the Galaxy are transients (Chen et al. 1997). The cause of the primary rise in outburst is most likely to be due to the disk thermal instability associated with the ionization of hydrogen and helium (Cannizzo et al. 1982; Mineshige, Wheeler 1989). The disk-instability model can also produce ultra-soft X-rays (\lesssim keV) and a strong correlation between the optical and X-rays in the outburst evolution (Mineshige et al. 1990a). There remain many interesting issues, however. The original disk-instability model (without irradiation) failed to produce a plateau in the V -magnitude like that 100–200 days after the peak in some systems. Although the disk-instability model can account for the beautiful exponential decays (Cannizzo et al. 1995), it has difficulty in producing the reflare (or secondary maxima) commonly observed in X-ray novae at 50–70 days after the main peaks. These were suggested to be due to irradiation of the outer portions of the disk (Mi-

neshige, Wheeler 1989; Mineshige 1994).

We examine here some of the aspects of the effect of irradiation on the propagation of heating and cooling waves in time-dependent disks that are unstable to the ionization thermal instability. Preliminary versions of this work were presented by Kim et al. (1994), Wheeler et al. (1996), Kim et al. (1996), and Wheeler (1997, 1999). A precursor to this work was that of Saito (1989), who was the first to consider time-dependent, irradiated disks with some simplifications (discussed later). Because this is the first study of time-dependent irradiation in black-hole disks, we concentrate on qualitative effects in systems like A0620–00. The model assumptions, including a prescription for treating irradiation, are given in section 2. The global effects of irradiation are presented in section 3 and the effects of irradiation on the thermodynamics of the heating and cooling instability are given in section 4. A discussion and conclusions are given in section 5.

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2. Disk-instability Model for Irradiated Accretion Disks

2.1. Irradiation

Two types of irradiation are considered: direct irradiation from the innermost hot disk and irradiation that might be reflected by a corona or disk atmosphere or chromosphere above the disk (Meyer, Meyer-Hofmeister 1994). The X-ray luminosity of the irradiation is given by

$$L_X(t) = \epsilon \dot{M}_{\text{in}} c^2, \quad (1)$$

where the efficiency ϵ is 0.057 and \dot{M}_{in} is the mass-accretion rate at the inner edge of the disk R_{in} , taken to be $3r_g$ as for a Schwarzschild black hole. We adopt a simple model for indirect irradiation by assuming that the luminosity of equation (1) effectively arises from a point at the center of the disk ($R = 0$). The indirect irradiation flux is then given by

$$F_i(R, t) = C_X \frac{L_X(t)}{4\pi R^2}, \quad (2)$$

where C_X is a constant. More sophisticated models incorporating radiative transfer would give $C_X = C_X(R, t)$ (see discussion in Tuchman et al. 1990), but such time-dependent radiative transfer is too difficult at this time.

For direct irradiation, we follow the prescription of Fukue (1992; see also Kusaka et al. 1970). This model assumes that the irradiation from a hot, geometrically thin annulus near the inside of the disk can be approximated by that from an infinitesimally thin, filled, uniformly radiating surface centered on the black hole. For regions in the disk at distances much larger than the radius of the annulus, this leads to the following expression for the total flux incident on one surface of the disk:

$$F_d(R, t) = (1 - A) \frac{L_X(t)}{4\pi R} \cdot \frac{d}{dR} \left[\frac{H(t)}{R} \right]^2, \quad (3)$$

where A is the X-ray albedo.

In practice, this irradiation is computed to arise from a disk with the radius of the inner zone and from a height above the disk plane equal to the disk height, H , of the innermost computed zone. The latter is a function of time and is used to compute the rays for direct irradiation of the outer disk.

Equation (3) is equivalent to that adopted by King et al. (1997, see their equation 6). It is smaller by a factor of H/R than the prescription used by van Paradijs (1996) for neutron-star systems. For simplicity, we take the X-ray albedo to be $A = 0.5$. De Jong et al. (1996) have deduced an albedo of $\sim 90\%$ for low-mass X-ray binaries, but the specific choice of the albedo does not appreciably affect the disk structure, since shadowing suppresses direct irradiation (see subsection 3.1). Equation

(3) shows that direct irradiation is a function of the gradient of the disk height, and is hence very sensitive to small variations in the disk profile. Analytic models assuming $d \ln H / d \ln R = \text{constant}$ may exaggerate the irradiation by minimizing the shadowing that may occur even in the steady state. They are certainly not adequate for time-dependent disks. The height of the region from which the irradiation arises varies with time and the disk height profile varies with both the irradiation and time. This makes the shadowing of the outer disk a complex, time-dependent phenomenon. We assume that no direct irradiation flux is added to the shadowed portions of the disk, while the indirect irradiation heats the whole disk in accord with equation (3).

2.2. Basic Equations and Boundary Conditions

Once \dot{M}_{in} , and hence $F_i(R, t)$ and $F_d(R, t)$, are determined, the irradiated flux is added to the flux generated internally in the disk by viscous heating through an implicit numerical method, and other physical variables are computed in each time step (see Tuchman et al. 1990 for details). A modification of the outer boundary condition by irradiation affects the whole vertical structure of the disk, including the mid-plane values (Tuchman et al. 1990). This is a reasonable first approximation, but could be improved by including more sophisticated radiative transfer effects. In this formulation, the irradiation fluxes are parameterized by two constants, C_X and A , which are, by assumption, independent of the radius and time.

The basic equations of the time-dependent thermally unstable accretion-disk model (for details, see Kim et al. 1992 and references therein) are the mass conservation,

$$2\pi R \frac{\partial \Sigma}{\partial t} = \frac{\partial \dot{M}}{\partial R} + 2\pi R S_0(R, t), \quad (4)$$

the angular momentum conservation;

$$2\pi R \frac{\partial (\Sigma \ell)}{\partial t} = \frac{\partial (\dot{M} \ell)}{\partial R} + 2\pi R S_0(R, t) \ell_K - \frac{\partial}{\partial R} (2\pi R^2 W), \quad (5)$$

and the vertically averaged energy equation;

$$\frac{C_p \Sigma}{2} \left(\frac{\partial}{\partial t} + v_R \frac{\partial}{\partial R} \right) T_c = Q_{\text{vis}}^+ + F_d + F_i - Q_{\text{rad}}^- + Q_{\text{dif}}, \quad (6)$$

where we use cylindrical coordinates (R, ϕ, z) , with the origin at the black hole and the z -axis perpendicular to the plane of the disk ($z = 0$). Also, Σ is the surface density, $S_0(R, t)$ is the mass source function defined by

$$\dot{M}_T(t) = \int 2\pi R S_0(R, t) dR, \quad (7)$$

with \dot{M}_T being the mass transfer rate from the companion star, $\ell = (GM r)^{1/2}$ is the specific angular momentum

of matter in the disk, ℓ_K is the specific angular momentum of the stream from a companion star, M is the mass of black hole, $W (= \int \alpha P dz)$ is the integrated viscous stress (with α and P being the viscosity parameter and pressure, respectively), T_c is the central temperature of the disk, $v_R = -\dot{M}/2\pi R\Sigma$ is the radial velocity of the matter, C_p is the specific heat with constant pressure, and Ω_K is the Keplerian angular frequency.

There are several terms on the right-hand side of equation (6). The disk heating in the non-irradiated disk is given by the viscous heating;

$$Q_{\text{vis}}^+ = \frac{3}{4}W\Omega. \quad (8)$$

The external-heating terms due to indirect and direct irradiation of the disk, F_i and F_d , are given by equations (2) and (3). In the present time-dependent study, we included irradiation as heating in the vertically averaged structure, and did not solve the vertical structure of the disk with irradiation input from above the surface (cf. Tuchman et al. 1990). The radiative cooling is given by

$$Q_{\text{rad}}^- = \sigma T_{\text{eff}}^4, \quad (9)$$

where σ is the Stefan–Boltzmann constant. Finally, Q_{dif} is the heat-diffusion term, defined by

$$Q_{\text{dif}} = \nu_{\text{th}} \frac{C_p \Sigma}{2R} \frac{\partial}{\partial R} \left(R \frac{\partial T_c}{\partial R} \right), \quad (10)$$

and $\nu_{\text{th}} (= 2W/\Omega\Sigma)$ is the thermal diffusivity. Here, we use the α formula with the same α as the viscosity (see below).

We employ the following viscosity parameter (α) prescription in a time-dependent manner (Mineshige, Wheeler 1989):

$$\alpha(R, t) = \alpha_0 \left[\frac{H(R, t)}{R} \right]^N, \quad (11)$$

where α_0 is a constant, and R and H are the disk radius and height. From the hydrostatic relation, $H = C_s/\Omega_K$, where C_s is the sound speed, we can rewrite equation (11) as

$$\alpha(R, t) = \alpha_0 \left(\frac{\mathcal{R}}{\mu} T_c v_\phi^2 \right)^{\frac{N}{2}}, \quad (12)$$

where \mathcal{R} is the gas constant, μ is the mean molecular weight, and $v_\phi(R) = R\Omega_K(R)$, the azimuthal velocity in the Keplerian disk. This implies that α is a function of the mid-plane or central temperature: $\alpha \propto T_c(t)^{N/2}$.

The boundary conditions are

$$\frac{\partial T_c}{\partial r} = 0 \quad \text{and} \quad \dot{M} = \dot{M}_T \quad (13)$$

at the outer edge and

$$T_c = W = 0 \quad (14)$$

Fig. 1. Opacities from Alexander et al. (1993), Cox and Stewart (1970), and Cox and Tabor (1976) for $\log \rho = -6.0$ to -8.0 at intervals of $\Delta \log \rho = 0.5$.

at the inner edge of the disk, where the mass-transfer rate from the companion star is taken to be constant in any particular model. Relativistic effects are not taken into account.

2.3. Methods of Calculations

The radially-dependent equations, (4)–(6), are solved with the equations obtained by integration of the vertical structure of disks,

$$f = f(T_c, \Sigma, R), \quad (15)$$

where the function f describes the disk parameters, such as the flux, Q_{rad}^- , viscous heating, Q_{vis}^+ (or W), disk height, H , central (mid-plane) density, ρ_c , and optical depth, τ .

We have taken the opacity from Alexander et al. (1983) for low temperatures ($\log T_c \lesssim 4.0$), and Cox and Stewart (1970) and Cox and Tabor (1976) for high temperatures ($\log T_c \gtrsim 4.0$) with mass fractions $X_H = 0.71$ and $X_{\text{He}} = 0.27$. The opacity for the intermediate temperature region is interpolated. The resultant opacity in the low-density region appropriate to our models in the outer disk is presented in figure 1, where the dominant effects are designated. We plot the opacity for $\log \rho_c = -6$ to -8 . The disk regions in which we are interested have a density distribution of $\log \rho_c = -6.2$ to -7.7 . The range, $\log T_c \sim 3.25$ – 3.55 , is the region dominated by molecular opacity. As shown in figure 1, the effect of molecules diminishes at $\log T_c \gtrsim 3.5$, depending on the density. At

$\log T_c \gtrsim 3.7$, H^- first becomes important. We define the domain between these two temperature regions as an intermediate state. The partially ionized region can reach well above 10^4 K, depending on the density. In terms of the effective temperature, the hot state is roughly $\log T_{\text{eff}} \gtrsim 10^{3.85}$.

Two regimes where changes in the physical conditions can affect the stability of the disk have been discussed in previous studies. One is the partially ionized, intermediate warm state centered at $\log T_{\text{eff}} \sim 3.8$, or, equivalently, $\log T_c \sim 3.7\text{--}4.1$ (Mineshige, Osaki 1985; Mineshige 1988; Kim et al. 1992). The other regime occurs in the temperature region centered at $\log T_{\text{eff}} \sim 3.4$ (or $\log T_c \sim 3.4\text{--}3.7$) in a marginally optically thin region (Cannizzo et al. 1982; Mineshige, Osaki 1983) dominated by molecular opacity, such as H_2O (Cannizzo, Wheeler 1984). These regions of variable opacity can lead to “metastable” or “stagnation” states in the evolution between the hot and cold stable states of the disk (discussed in subsection 4.2). Ionization stagnation is explicitly displayed in models for the outburst rise by Mineshige (1988) and Kim et al. (1992). The possible significance of the molecular opacity to the disk-instability model was raised by Cannizzo and Wheeler (1984), but no explicit presentation has been provided.

The basic implicit numerical code used to compute time-dependent disk-instability models is that given by Mineshige (1986; see also Mineshige, Wheeler 1989; Kim et al. 1992). The present disk-instability models have been improved compared to those presented by Mineshige and Wheeler (1989), in which an unreasonably small disk size (3.16×10^{10} cm) was adopted so as to avoid numerical instabilities and in which irradiation is ignored. These models represent an implicit computation of 21 radial zones separated by radial mesh points at fixed spacings of $\log \delta R \sim 0.22$, from $\log R_{\text{in}} = 6.5$ to $\log R_{\text{out}} = 11.0$. This zoning is adequate to reproduce the qualitative features of the expected outburst light curves for our adopted prescription for the viscosity parameter (see below). It is clearly not sufficient to provide a fully satisfactory model of a time-dependent irradiated disk. These models represent the first study of both the radial and time-dependent nature of irradiated disks around black holes where the central star cannot provide a source of flux. They are presented as a step beyond the single irradiated annuli studied by Tuchman et al. (1990) and Mineshige et al. (1990b) and as a step toward the ultimate solution of time-dependent irradiated disks. They should not be viewed as full disk models, but as collections of coupled annuli that, nevertheless, give some qualitative insight into the physics of irradiated, time-dependent accretion disks.

2.4. Model Parameters

We choose a $4M_\odot$ black hole with a companion of $0.27M_\odot$ as being representative of A0620–00 (Marsh et al. 1994 and references therein). This choice, with the observed orbital period of 7.75 hr, gives the following disk model parameters: $R_{\text{in}} = 3.6 \times 10^6$ cm, binary separation 2.24×10^{11} cm, Roche lobe radius of the primary system 1.83×10^{11} cm and the disk radius 10^{11} cm (Frank et al. 1992; Paczyński 1977). Both the inner and outer boundaries of the disk are held fixed. We tested a range of values of R_{out} , and found that the qualitative behavior is similar for outer-disk radii in the range 5×10^{10} cm $< R_{\text{out}} \lesssim 1.3 \times 10^{11}$ cm, the tidal radius of the disk. We choose the mass-transfer rate from the companion star to be $\dot{M}_T = 3.16 \times 10^{16}$ g s $^{-1}$. This is larger than that given by McClintock and Remillard (1986), but within the observational uncertainty (see the discussion in McClintock et al. 1995). The transferred mass from the companion is initially accumulated in the outermost disk ($6 \times 10^{10} - 10^{11}$ cm) and then diffuses and accretes inward.

The outburst recurrence time in these models is about 3–4 years. This is too short for most observed systems, but does not affect the qualitative thermodynamics of the irradiation, which is the current focus. The recurrence time would be longer if the transfer rate were smaller (see also King et al. 1997). For this relatively short recurrence time, the inner disk does not have time to return entirely to the cool state. The mass-flow rate in the inner disk, and hence the generated radiation, does drop by many orders of magnitude (about 7) from the peak, which is sufficient to capture the essence of the variable production of irradiation and how it reacts with the outer disk during the phases of heating and cooling.

Given the limitation of the small number of zones, we have checked the basic behavior of the code by computing the speed of propagation of the cooling wave. Cannizzo et al. (1995) express the speed of the cooling front as

$$V_{\text{front}} = \frac{2}{3} \alpha_0 \left(\frac{H}{R} \right)^{N+1} \frac{C_s}{w} R, \quad (16)$$

where $H = C_s \Omega$ is the vertical scale height at radius R , C_s is the sound speed, Ω is the Keplerian angular velocity, w is the width of the cooling front and the viscosity parameter is given by $\alpha = \alpha_0 (H/R)^N$. The results of our numerical models are in reasonable agreement with this expression for our assumed α prescription and logarithmic zoning. In the first 40 days after the maximum, the velocity of the cooling wave, V_{front} , is almost the same in both the irradiated and the non-irradiated models, but slightly slower in the irradiated case. At later times, the cooling-wave speed in the irradiated model is somewhat faster than that for the non-irradiated case, although both become very slow, less than 10^3 cm s $^{-1}$ after 100 days. The overall shapes of the velocity profiles are rather similar in both models.

Although a certain range has been explored, we have not undertaken a systematic parameter study of these computationally expensive irradiated models. Varying C_X , α_0 , and N gives different slopes of the light curves in the decay and different intervals between the peaks of the optical activity and the inner mass flow rate. If the indirect irradiation efficiency parameter, C_X , is sufficiently large, $C_X \sim 10^{-2}$, the models will be heated to a permanently hot steady state (see van Paradijs 1996). To approximately reproduce the observed light curve of A0620-00, we adopt $N = 2$, $\alpha_0 = 10^3$, and $C_X = 1.85 \times 10^{-4}$. For the chosen parameters, the outbursts in these models always begin in the outer disk. This determines the systematics of the overall outburst. In particular, for systems with a fast rise and exponential decline, the optical outburst is predicted to arise well before the increase of the mass flow rate in the inner disk that might be associated with harder flux.

We have computed several repeated outbursts in each model to achieve an approximate steady state. Our current models with irradiation show an outburst cycle of ~ 4 yr. We did not attempt to reproduce longer recurrence times in this study, since our main current consideration concerned the effects of irradiation on the outburst, itself. In the disk-instability model, the recurrence time scale is related to α , \dot{M}_T , R_{out} and the masses of the binary components, M_1 and M_2 (e.g., Cannizzo et al. 1988; Mineshige, Wood 1989). In principle, the easiest way to achieve a longer recurrence time would be to decrease \dot{M}_T . While the value we have chosen here, $\dot{M}_T = 3.16 \times 10^{16} \text{ g s}^{-1}$, is plausible, it could be a factor of 10 smaller and still agrees with the observational constraints on, e.g. A0620-00.

3. Outburst Evolution

3.1. Light Curves

The optical light curves are computed by assuming that each disk annulus radiates as a black body at temperature $T_{\text{eff}}(R, t)$ and summing the emission in the V band over the annuli. The resultant model optical light curves are presented in figure 2 together with observations of A0620-00. (The data for A0620-00 is plotted 1.5 mag brighter than the observed value for clarity of presentation.) The model with both direct and indirect irradiation gives an optical light curve with a fast rise, a maximum, and a nearly exponential decline. As can be seen from figure 2, the direct irradiation has virtually no effect on the light curve. The modest indirect irradiation we have invoked gives a slight, but noticeable flattening of the light curve. More severe indirect irradiation would cause this flattening to be more extreme. For the irradiation parameter $C_X \sim 10^{-2}$, the disk would be stabilized. The nature of this irradiation stabilization

Fig. 2. Model light curves on the decline from the maximum along with the optical observations of A0620-00 (Webbink 1978 and private communication). The model optical maximum is defined as day zero. The data for A0620-00 is plotted 1.5 mag brighter than observed for clarity of presentation and their phase is set by assuming coincidence of the observed soft X-ray peak and the model optical maximum (see text). The model with both direct and indirect irradiation fits the observed optical light curve of A0620-00 with an assumed inclination of 60° and a distance of 400 pc.

will be explored in future work.

Note that our calculations never run into a radiation pressure-dominated regime nor an optically thin one, probably because the peak luminosity remains a slightly below that at which the radiation pressure dominates over the gas pressure.

3.2. Mass Flow

The model accretion rate through the inner boundary (R_{in}) of the disk is presented in figure 3. Note that since the mass flow rate depends sensitively on R_1 as $\dot{M} \propto R_{\text{in}}^{2.6}$ and since R_{in} is much larger than that of the marginally stable circular orbit ($3r_g$) in our calculations, due to the crude zoning, our quiescent mass-flow rate is much larger than that given by the fine-mesh calculations of Ludwig et al. (1994). However, the qualitative behavior should be adequately captured by these models.

The indirect irradiation leads to a lower \dot{M}_{in} in quiescence due to a higher depletion of the disk mass during outburst. The mass-transfer rate through the inner edge of the disk determines one component of the soft X-ray flux. In the current models, \dot{M}_{in} does not begin to rise

Fig. 3. Time-dependent evolution of the innermost accretion rate (\dot{M}_{in}) for cases with no irradiation (short dash), direct irradiation only (long dash), indirect irradiation only (dot-dash), and both direct and indirect irradiation (solid line).

until 13.6 d before the primary optical peak, over two weeks since the start of the optical display. The peak in the mass flow rate occurs about 20 d after the primary optical maximum and about 30 d after the first rise in the inner mass-flow rate. Qualitatively, these models predict that the optical flux from the outer disk should rise before any activity from the inner disk, soft X-rays from the inner optically thick disk or hard or soft X-rays from any coronal emission. This is consistent with the results of multi-wavelength observations of Nova Muscae (see Lund 1993). A prediction of the absolute times of peak emission in any band is more uncertain in the absence of reliable models for the time-dependent power-law emission.

3.3. The Shadow Effect

The non-monotonic distribution of the disk height causes the middle portions of the disk to block the flux of direct radiation emitted from near the inner edge of the disk, and thus prevent the direct irradiation of outer parts of the disk. The shadowing of the outer disk is determined by the radial distribution of the opening angle, $\tan \theta = H/R$, assuming the radiation to arise from an inner region of small H and R . A region with a given value of H/R will thus be shadowed by a region at smaller R with larger H/R . Outer regions with small H/R in the wake of an inward-propagating cooling wave will clearly be in the shadow of inner, hot regions.

Fig. 4. Shadow effect for the direct irradiation. The hatched part in the figure signifies where a given zone at a given time is in the shadow of some interior zones that are exposed to direct irradiation.

Some systematics of the shadowing process are illustrated in figure 4. The specific details reported here may be subject to the effects of finite zoning. The qualitatively significant results arise from comparing models with and without irradiation.

Figure 4 shows that the innermost disk ($\lesssim 2 \times 10^7$ cm) is always exposed to the direct irradiation. Because the outburst is initiated about 25 d prior to the optical peak, more extensive portions of the disk are exposed to direct irradiation. After maximum light, the outer disk ($> 2 \times 10^8$ cm) is essentially totally blocked from direct irradiation by the inner disk. With the shadowing effect, therefore, the existence of a mechanism which can reflect the radiation from the inner region is important in the current models if there are to be any effects of irradiation in the outer disk.

3.4. Amplitude of Disk Irradiation

The disk heating in the non-irradiated disk-instability model is given by the viscous heating, Q_{vis}^+ [see equation (8)]. In the presence of irradiation, the total heating is described by

$$Q_{\text{tot}}^+ = Q_{\text{vis}}^+ + F_{\text{i}} + F_{\text{d}}. \quad (17)$$

The presence of irradiation affects the disk structure, and hence the contribution from Q_{vis}^+ , itself, is altered. The resultant contribution of the irradiation to the total disk heating is presented in figure 5 for an inner, a middle, and

Fig. 5. Contribution of irradiation in the disk evolution. For illustration, we choose four zones: 5 (short dash), 16 (dot), 18 (dash dot), and zone 20 (solid line). The contribution of both direct and indirect irradiation is plotted with respect to the total heating in the upper panel. In the lower panel, we present the ratio of direct to indirect irradiation. The direct irradiation shuts off its effect about 20 d after outburst maximum (day zero) due to shadowing (see figure 4).

an outer region with the model optical maximum again being defined as day zero. In the inner region ($\lesssim 10^8$ cm), the irradiation is negligible throughout the entire evolution. The direct component is about 10 % of the indirect component in the inner region, but the total irradiative heating does not contribute substantially compared to the viscous heating. In the middle of the disk ($\sim 10^{10}$ cm), the contribution of the irradiation is small, but finite. Direct irradiation is appreciable only during the early decay, and is truncated by 20 d due to shadowing. In the outer disk ($\sim 6 \times 10^{10}$ cm), the contribution of the irradiation becomes important. Around maximum, the viscous heating dominates the irradiation. After the cooling wave propagates into the outer region, and this region cools, the irradiation heating, due entirely to indirect irradiation because of shadowing, dominates over the viscous heating. This is shown by the rapid increase in the ratio of irradiation heating to the total heating about 5 d after the maximum in figure 5. The indirect irradiation heating then decreases along with \dot{M}_{in} as the cooling wave propagates inward. The behavior of the outer regions under the effects of radiation is more complicated

Fig. 6. Time-dependent evolution of the disk mid-plane temperature during the early decay phase (upper panel) and the corresponding states of the gas (lower panel) for the model with no irradiation. “HOT” denotes the hot thermal equilibrium states and “MOLECULE” represents the molecular stagnation stage (see text and figure 1).

than reflected in figure 5.

3.5. Global Effects of Irradiation on Decay

The influence of mild irradiation is not restricted to a small modification of the light curves. Rather, it causes crucial changes in the local disk structure, as illustrated in figures 6 and 7. The upper panel of figure 6 presents the time-dependent evolution of the disk central temperature in the model *without irradiation*. This model reproduces the typical decay phase, as shown in numerous other studies of the disk instability. As the cooling wave propagates from the outer to inner radii, the disk makes the transition from the hot to cool state from larger to smaller radii. Note the sharp edges of the rapid transition from the hot to cool state, which represent the cooling wave propagation. The lower panel of figure 6 shows the opacity domains through which the disk evolves. It is basically only in the hot, ionized or cool, molecular states with very rapid transitions between them. This is because in the non-irradiated model, or in the model with only direct irradiation and shadowing, the stagnation phase is absent.

Figure 7 displays the time-dependent disk evolution in the decay phase for the irradiated disk-instability model

Fig. 7. Time-dependent evolution of the disk mid-plane temperature during the early decay phase (upper panel) and the corresponding states of the gas (lower panel) for the model with both direct and indirect irradiation. “HOT” denotes the hot thermal equilibrium states “Partial Ionization” means that the disk is in the metastable ionization stagnation stage, and “MOLECULE” (or “M”) represents the molecular stagnation stage. “Intermediate State” (or “i”) denotes the intermediate state between the molecular and partially ionized region (see text and figure 1).

with both direct and indirect irradiation. The upper panel of figure 7 shows the evolution of the disk central temperature, T_c . The squares denote the time at which the direct irradiation is terminated at a given radius as it falls into the shadow of interior regions. With irradiation, the evolution is much more complex, as illustrated in figure 7. At 14.2 d, the region at $\sim 4 \times 10^{10}$ cm reaches the critical surface density for a thermal instability and begins the rapid cooling that characterizes the propagating cooling front of figure 6. The same transition is seen at $\sim 2 \times 10^{10}$ cm after 41.6 d and at $\sim 1 \times 10^{10}$ cm after 88 d. Rather than dropping directly to the cool state, however, these regions linger in an intermediate “metastable” stagnation state. This is obviously due to the effects of irradiation, since this behavior is absent in figure 6, the effects, however, are rather subtle and involve global coupling in the outer regions.

Throughout the disk evolution from quiescence to outburst, the outermost disk (around 10^{11} cm) stays in the cold, neutral state at $\log T_{\text{eff}} \lesssim 3.4$, or $\log T_c \lesssim 3.55$. Af-

ter maximum light, the cooling wave is initiated around 6×10^{10} cm. The resulting outward diffusion of the surface density results in a small increase in the temperature at larger radii. The molecular stagnation (discussed in subsection 4.2) and consequent higher temperature of the outer regions due to the indirect irradiation affects the behavior of the inner regions. The regions just interior to the molecular-stagnation region can not simply drop into the cool state, because there is now some resistance to the outward diffusion of matter that is attendant to the non-irradiated cooling wave. Instead, the decline in temperature after the onset of the cooling instability is halted in the nearly constant temperature intermediate ionization stagnation state, as shown in figure 7.

After the outburst maximum, the X-ray novae have exhibited additional features in the decay, such as “reflares” 50–80 d after the maxima, the “second maxima” a few hundred days later, and subsequent “mini-outbursts” before returning to quiescence. The reflare is common in many X-ray novae throughout a variety of wavelengths: optical (A0620–00), UV (Nova Muscae 1991, GRO J0422+32) and soft X-rays (A0620–00, Nova Muscae 1991, GRO J0422+32). As shown in figure 7, one portion of the cooling disk undergoes a heating instability that takes it back to the hot, ionized state, thus producing a transient increase in the light curve (see figure 2). This might be associated with the reflare, since the optical feature is consistent with Kuulkers (1998), who summarized the data for the reflare in A0620–00 in both optical and X-rays (with some delay). It is not clear whether the optical feature in our models has some physical basis or whether it is an artifact of the crude zoning. Our models do not provide any modulation of the inner mass flow and hence of the soft X-rays at the time of the reflare. Further studies of these issues are needed.

4. Irradiation and Thermodynamics

4.1. The Limit Cycle

Figure 8 presents the actual track of the evolution of the surface density and mid-plane temperature T_c , the so-called thermal limit cycle (for reviews, see Osaki 1989; Cannizzo 1993) for various models. The top panels in figure 8 present the evolution of the region around $1 - 2 \times 10^{10}$ cm (panel a) and around $4 - 6 \times 10^{10}$ cm (panel c) for the case with both direct and indirect irradiation. The open circles show when the region is subject to direct irradiation and the closed circles show when the region is shadowed from direct irradiation at the indicated times. For the same two regions, the lower panels in figure 8 present models corresponding to no irradiation (long dash), direct irradiation only (short dash), and indirect irradiation only (dotted line). The model with only indirect irradiation shows very similar evolution to

Fig. 8. Thermal limit cycles in the (Σ, T_c) plane for annuli at 2×10^{10} cm (left panels) and 6×10^{10} cm (right panels) with both direct and indirect irradiation during the decay phase from the outburst maximum to the quiescent minimum state. In the upper (a) and (c) the open and filled circles represent epochs when the zone is and is not subject to direct irradiation, respectively. The outer region, at 6×10^{10} cm, receives no direct irradiation due to the blocking by the inner disk and hence the shadowing is in effect from the outburst maximum until the disk returns to the cool quiescent state (see figure 4). The numbered epochs correspond to times after maximum in days as follows: 1 – 0.94; 2 – 3.5; 3 – 4.7; 4 – 6.9; 5 – 14; 6 – 15; 7 – 21; 8 – 41.6; 9 – 41.9; 10 – 44; 11 – 53; 12 – 54; 13 – 55; 14 – 57.2; 15 – 57.4; 16 – 57.5; 17 – 70; 18 – 88.2; 19 – 88.4; 20 – 94; 21 – 130; 22 – 211. In the lower (b) and (d) the trajectory of the annulus evolution is illustrated for three cases: non-irradiated (dotted line), directly irradiated only (dashed), and indirectly irradiated only (solid line) models.

the full model. There are three distinctive stages to the full model:

(1) In the region from $1 - 2 \times 10^{10}$ cm, the temperature stops declining and then climbs back up to the hot state again (figure 8a, time steps 9–14).

(2) On the second decline from the hot state the disk temperature *stagnates* at $\log T_c \sim 3.8$ –4.1 (figure 8a, time steps 16–19) in the partially ionized region (see figure 7 and subsection 4.2), prior to the final downward transition to the cool disk. We call this phase an “ionization stagnation.”

(3) In the outer region, $\sim 4 - 6 \times 10^{10}$ cm, the disk shows a similar stagnation, but it occurs at $\log T_c \sim 3.4$ –3.6 (figure 8c, time steps 3–7) in the molecular opacity-dominated region (see figure 7 and section 4.2). We call this phase “molecular stagnation.”

Figure 8 shows that the stagnation behavior on the decline is not present in the non-irradiated or only directly irradiated models. Even with modest indirect irradiation this stagnation behavior can affect the outburst and cooling of the disk.

4.2. Stagnation Phenomena

The disk is in thermal equilibrium when the viscous heating, Q_{vis}^+ [equation (8)], balances the radiative cooling, Q_{rad}^- [equation (9)], or $Q_{\text{vis}}^+ = Q_{\text{rad}}^-$. In the thin disk approximation (e.g., Shakura, Sunyaev 1973), the emergent heat flux radiated vertically in the plane-parallel approximation is given by $F(z) = 4\sigma T_c^4/3\tau$. Since $\tau = \Sigma\kappa$ for the Rosseland mean opacity κ and the surface density is $\Sigma = 2\rho H$ for density ρ and disk height H , one can write

$$T_{\text{eff}}^4 = \frac{8T_c^4}{3\kappa\Sigma}. \quad (18)$$

Since the sound speed is $C_s = (\mathcal{R}T_c/\mu)^{1/2}$, where \mathcal{R} is the gas constant and μ is the molecular weight, and $H = C_s/\Omega$, the energy equation in thermal equilibrium becomes

$$\frac{3}{2}\alpha\Sigma\left(\frac{\mathcal{R}T_c}{\mu}\right)\Omega = \frac{3}{2}\alpha\Omega\frac{P}{\rho} = \frac{4acT_c^4}{3\kappa\Sigma}, \quad (19)$$

where $\sigma \equiv ac/4$ and $\kappa = \kappa(T_c, \rho_c)$. Note that, in the disk-instability model which we present here, α is also a function of T_c [see equation (11)]. In various physical regimes, the opacity can be expressed as

$$\kappa = \kappa_0\rho_c^m T_c^n, \quad (20)$$

where κ_0 is a constant and $m \geq 0$, but n can be negative or positive (see figure 1).

The possibility of stagnation in a partially ionized region was first pointed out by Meyer-Hofmeister (1987) and elaborated by Mineshige (1988). For the case of a departure from thermal equilibrium, the energy equation, equation (6), can be written as

$$\frac{C_p\Sigma}{2}\frac{\partial T_c}{\partial t} = Q^+ - Q^-, \quad (21)$$

where C_p is the specific heat at constant pressure, and Q^+ and Q^- are the total heating and cooling rates. The rate of change of the temperature can be decreased by increasing the specific heat, or by decreasing the departure from thermal equilibrium, $Q^+ \sim Q^-$, or a combination of both. The essence of the higher temperature stagnation is that the specific heat increases in the regions of partial ionization and if the structure is not too far from thermal equilibrium, the thermal time scale is increased, leading to a “metastable” intermediate temperature state. In the rise, the partially ionized region is near the “knee” in the instability curve, where $Q^+ \sim Q^-$ and the effect can be pronounced.

Mineshige (1988) showed that the onset of the heating can consist of a “warming wave” which raises the disk from the cold state to this intermediate metastable state. This state is ultimately unstable and a full heating wave causing the transition to the hot state finally

ensues. In the decline, the partial ionization state is further from the equilibrium curve, and although the specific heat increases, the cooling term, Q^- , dominates the heating term, and there is little effect of the specific heat as the disk drops into the cool state. As a result, stagnation has not been manifested in the decline in previous disk-instability models. As the current models illustrate, irradiation can enhance the heating term, Q^+ , in the decline, thus restoring an approximate balance, $Q^+ \sim Q^-$. In this situation, the effect of an increase of C_p can be seen as “stagnation” in the decline as well. This can cause the disk to linger in the intermediate-temperature state. It eventually makes a downward transition into the cool state.

At $\log T_c \lesssim 3.5$ – 3.6 , the molecular absorbers become important and at $\log T_c \sim 3.4$, H_2O dominates. The low-temperature opacity is dominated by grains as grain condensation begins at $\log T_c \lesssim 3.25$. These various effects change the slope of the opacity at about $\log T_c = 3.25, 3.4$ and 3.5 – 3.6 (figure 1). As a result, when the disk passes through $\log T_c = 3.6$, the opacity suddenly *increases* with *decreasing* temperature [see equation (18)]. As a result, the thermal time scale for the downward transition will be increased.

5. Discussion and Conclusions

The purpose of this paper was to investigate some of the basic time-dependent effects of irradiation. We show that even relatively mild irradiation from the inner disk can affect the overall disk evolution. Mild irradiation can affect the limit cycle by causing portions of the disk to linger in the intermediate-temperature metastable “stagnation state.” In contrast to the non-irradiated disk-instability models, the irradiated models show a wave of cooling to the stagnation state that propagates inward in advance of cooling to the low, molecular-dominated state. It is not clear that this is simply a result of heating of the disk, since the non-linear interplay of the zones seems to have an important role. The issue of how a particular portion of the disk responds to irradiation by tracking along the metastable state is a rather subtle thing involving complex interactions among the different portions of the disk. Due to shadowing, the effect of direct irradiation terminates as each annulus of the disk enters the stagnation state. The net result of the irradiation is a slower decay of the light curve.

When an outburst is initiated, the outer disk is exposed to direct irradiation, but that radiation is feeble until the heating wave reaches the inner disk. Soon thereafter the outer disk is shadowed by a swelling of the middle of the disk. Direct irradiation does not play an important role in the behavior of the outer disk either during and after the maximum of the outburst, but can affect the middle of the disk during the outburst. Since it is by assumption

not influenced by shadowing, the indirect irradiation affects the overall disk evolution, including the outer disk, throughout its evolution.

As stated in the introduction, this is the first study of time-dependent irradiation of black-hole accretion disks. For this reason and technical issues related to code instability, we have presented models with rather crude zoning. These models are a major step beyond the one-zone models of Tuchman et al. (1990), but only a step toward the goal of self-consistent, resolved irradiated models. Beside the rather crude zoning, other restrictions are rather simplified models for the irradiation that make no attempt to do realistic radiative transfer and, of course, continued uncertainty about the physics of disk viscosity and the role of disk evaporation. In the context of these general limitations and uncertainties, these models do give some insight into the issues of time-dependent irradiation. They incorporate sufficient physics to capture the non-monotonic nature of the disk profile, a critical issue in considering shadowing. Given this context, there are some qualitative aspects of these models that are reasonably trustworthy:

- The direct irradiation of the disk by inner layers where the soft X-ray flux is generated is found to have only a small effect on the outer disk because of shadowing.
- Mild indirect irradiation that flattens, but otherwise does not affect the light curve substantially, still has interesting non-linear effects on the structure of the disk as heating and cooling waves propagate.

Any quantitative results (e.g., front propagation velocity, the detailed shape of the light curves, etc) may change when fine-mesh calculations are performed. Specifically, the calculated reflare-like optical events may or may not be a calculation artifact. We need fine-mesh calculations to confirm the reflare-like behavior of the irradiated disk, but the basic phenomenon of radiation-induced stagnation allowing a temporary return to the hot state is physically plausible. It is not clear how such a phenomenon could lead to a modulation of the soft X-rays, a defining feature of the observed reflare.

Saito (1989) considered the case of a central star that illuminated the disk using a simple one-zone model for the vertical disk structure, and thus did not properly consider the non-(thermal)-equilibrium states of the disk. The irradiation from a central source is larger than what we consider here by a factor of H/R (cf. subsection 2.1, van Paradijs 1996). In addition, Saito neglected a term equivalent to $[d \ln H / d \ln R - 1] = (R/H)[d(H/R)^2 / dR]$ [see equation (3)], that measures the shape of the disk profile. He took the angle of incidence of the radiation, $\sim H/R$, and the flux generated by accretion to be constant. Despite these differences and approximations,

Saito's results are qualitatively similar to ours. With a large accretion rate (10^{17} g s^{-1}) and the various approximations that enhance the irradiation, Saito found that the inner disk was always maintained in a permanently hot state. The outer disk, however, underwent a thermal instability that began in the outer part of the disk. Saito notes that prior to the outburst the outer disk is in the shadow of the inner disk.

The disk instability due to the ionization of hydrogen and helium remains the most plausible cause of the outburst of the black-hole candidate X-ray novae. For the orbital periods and mass-transfer rates inferred from observations, the disk is predicted to be unstable. A steady state is very unlikely. Because the direct component of irradiation comes from a surface of the disk that is nearly in the plane of the outer disk, and hence presents a small solid angle, disks around black holes receive relatively little direct irradiation (Cannizzo 1994; King et al. 1997; Cannizzo 1998; Dubus et al. 1999). Gradients in opacity, the viscosity, the effect of the inner boundary condition, and of time-dependent structure, all serve to diminish the direct irradiation even further. Whether a source of "indirect" irradiation as we have modeled it here can have a more severe effect, including stabilizing the disk (van Paradijs 1996), requires further investigation.

The value of the indirect irradiation parameter, $C_X \sim 10^{-4}$, was chosen in conjunction with other parameters in these models to reproduce the approximate slope of the optical light curve of A0620-00. We find that this level of irradiation does not significantly affect the luminosity at maximum, but does make the disk brighter by about 0.5 mag at about 50 d after maximum. Cannizzo (1998) has estimated that A0620-00 might be brighter by about 1 mag at maximum light than his standard non-irradiated models. This difference may be within the model uncertainties, but, if ascribed to irradiation, it could be accounted for with a modest increase in C_X .

We have shown that even modest levels of irradiation can influence the thermodynamics of the disk and affect its evolution. It is very important to consider indirect as well as direct irradiation of the disk, since the effect of the latter is severely constrained by shadowing.

Finally, we can consider how our results might change if the quiescent disk has a hole in the middle because of gas evaporation and resultant advection-dominated flow. Such a hole will change, for example, the quiescent surface-density distribution, thereby affecting the ignition time and radius (Hameury et al. 1997). Also, the disk should expand during quiescence because of continuous input of angular momentum by the incoming stream, while the mass increase will be modest (Mineshige et al. 1998). Once an outburst occurs, however, the entire disk evolution will be totally controlled by the propagation of the heating front, except at the very early rise phase, so that our conclusions will not change significantly.

We are grateful to John Cannizzo, Wan Chen, Neil Gehrels, Robert Hjellming, Shunji Kitamoto, Masaaki Kusunose, Jean-Pierre Lasota, Don Lamb, Jeff McClintock, Yoji Osaki, Edward Robinson, Divas Sanwal, Chris Shrader, Ethan Vishniac, and the GRO BATSE NASA/MSFC team, especially to Alan Harmon, William Paciesas, and Jan van Paradijs for fruitful discussions and to the anonymous referee for comments that helped to clarify the manuscript. We acknowledge the optical light curves of A0620–00 sent by Ron Webbink and the optical light curves of Nova Sco 1994 sent by Jerome Orosz and Charles Bailyn. S.-W. K. gives special thanks to Seok-Jae Lee, Gei-Youb Lim, Myung-Shin Chung, Kyong-Sei Lee, and Kwang-Souk Sim of the Nuclear Physics Lab of Korea University for their hospitality and help with computation and plots during his stay in Seoul, Korea. S.-W. K. thanks to Hwankyung Sung, Won-Kee Park, and Hak Min Kim for helping to prepare figures and Seung Soo Hong for his encouragement during the first year of the KRF fellowship. Numerical calculations were performed at the High Performance Computing Facility (HPCF) of the University of Texas at Austin. This research was supported in part by NSF Grants AST 9115143, AST-9218035 and AST-9528110, NASA Grant NAGW 2975, a grant from the Texas Advanced Research Program (JCW), and the Grants-in Aid of the Ministry of Education, Science, and Culture of Japan (10640228, SM). S.-W. K. wishes to acknowledge the financial support of the Korea Research Foundation made in the program Year 1997–1998 and Year 1998–1999.

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